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THE LIAR PARADOX AND MANY-VALUED LOGIC

BY S. V. BHAVE

A philosophically satisfactory resolution of the paradox of the Liar and the strengthened Liar sentences needs an extension of logic to incorporate logical principles governing inferences when expressions like 'most probably true' etc. are involved. This is irrespective of whether we choose to call such expressions 'truth values' or 'truth-value gaps'.

I. THE PARADOX

The sentence

(S) This sentence is false

is paradoxical, because if we suppose (S) is true, then what it says is the case; so it is false. On the other hand, if we suppose (S) is false, then what it says is not the case; so (S) is true. So (S) must be neither true nor false.

Another way of describing the paradox is as follows:
Since the words 'This sentence' in (S) refer to the whole sentence (S), what (S) says is

(M) 'This sentence is false' is false.

If we assume that (S) is true, what it says is the case, that is, (M) is true, that is,

(S_1) ('This sentence is false' is false) is true

is a true sentence. So from (S_1) we can derive another true sentence by replacing the words 'is false, is true' with which it ends by the words 'is false'; that is:

'This sentence is false' is false

that is, '(S) is false' is a true sentence. This contradicts the assumption that (S) is true.

On the other hand, if we suppose (S) is false, then (M) must be false, so the sentence

(S_2) ('This sentence is false' is false) is false

is a true sentence. So replacing the words 'is false, is false' with which it ends by the words 'is true' we can get another true sentence, namely,

'This sentence is false' is true

so '(S) is true' is a true sentence which contradicts the assumption that (S) is false.

It is clear that the paradox of the Liar sentence involves the two presuppositions

1. Every meaningful declarative sentence has only one of the two truth values 'true' and 'false'.
2. If in a true sentence ending in 'is false, is true' we substitute for them the words 'is false' we get another true sentence; similarly if in a true sentence ending with the words 'is false, is false' we substitute for them the words 'is true', we get another true sentence.

It is clear that any proposed resolution of the paradox of the Liar sentence must imply giving up these presuppositions through adoption of some modification of our semantic concepts or principles of inference. Several such modifications have been proposed in the past, but none of these appears to be philosophically satisfactory. We shall first examine the major past proposals and consider in what respects they are philosophically unsatisfactory; later from some insights contained in these past proposals, we shall attempt to develop another solution of the Liar paradox which should be philosophically rather more adequate and satisfactory.

II. PREVIOUS PROPOSALS

The simplest solution of the Liar sentence would be to assume that since (S) does not appear to be either true or false, it must have some third truth value other than 'true' and 'false'. Bochvar (1939) had proposed the adoption of a three-valued logic in which the third truth value, 'paradoxical', is to be taken by recalcitrant sentences like (S). The proposal is not very satisfactory, for it raises the problem as to what truth value is to be ascribed to the strengthened Liar sentence

This sentence is paradoxical

which would appear to be paradoxical, if true, and true, if paradoxical. What is perhaps a more serious difficulty in accepting this as a philosophically satisfactory solution of the Liar paradox is that it is not clear *why* (S) is 'paradoxical' instead of being 'true' or 'false'. There seems to be no reason for assuming a third truth value like 'paradoxical' other than that sentences like (S) are neither true nor false; and to call (S) 'paradoxical' seems little more than simply to *label* the offending sentence in a way that is not really explanatory.

Another proposed solution of the Liar paradox is to assume that 'true' and 'false' mean different things depending on the level of the sentence in which they occur; in other words, that Tarski's ideas of the hierarchy of languages are applicable to natural languages in which the Liar sentence occurs. This suggestion is not acceptable, for Kripke (1975) has convincingly pointed out that ordinary ascriptions of truth and falsity in sentences in natural languages cannot even be assigned implicit levels. Burge (1979) has suggested that there is an indexical element at work in creating the Liar paradox, and that the same sentence 'that lacks truth_i conditions may, indeed will, have truth_k conditions, and can be evaluated as true_k or false_k'. But what Kripke (1975) said about implicit indices marking levels of sentences could be said about Burge's (1979) implicit indexical indices as well: 'Unfortunately this picture seems unfaithful to the facts. . . . If someone makes such an utterance as (1), he does *not* attach a subscript, explicit or implicit, to his utterance of "false" '. Kripke (1975) has suggested that the Liar sentence has *no* truth value, and that only *grounded* sentences can be either true or false; the Liar sentence is not *grounded*. Gupta (1982) holds that we have a 'rule for truth' which is a rule for revision of truth value, and the liar sentence, if classed as true at one stage, will be classed as false at the next, never stabilizing at any stage. No doubt, the ideas of 'groundedness' and 'rule for truth' contain valuable insights; but they do not resolve the Liar paradox. To hold that the Liar sentence has *no* truth value is not a satisfactory solution, for, as Burge (1979) has pointed out, the paradox re-appears with the strengthened Liar sentence. Furthermore, unless we were to hold the Liar sentence to be quite meaningless, it becomes a very mysterious situation indeed if a sentence which says nothing beyond that it has a truth value, really has just *no* truth value.

In order to develop a *philosophically* more satisfactory solution, we must try to combine the important insights in the different approaches discussed above. In the first place, we must see whether truth values other than 'true' and 'false' can be identified with terms in actual use in natural languages and are attributable to the Liar sentence and the strengthened Liar sentence. Secondly, several authors have pointed out the importance of *context* in creating paradoxes. Thus Kripke (1975) has said 'many, probably most, of our ordinary assertions about truth and falsity are liable, if the empirical facts are extremely unfavourable, to exhibit paradoxical features'; and Burge (1979) has talked of 'contextual applications of the indexical "true"'. It is in the context of bivalent logic that the Liar and the strengthened Liar sentences appear so paradoxical. How the situation would appear in the context of a suitable many-valued logic *actually in use in natural languages* is well worth examining.

In the next section we shall first examine what conditions must be satisfied by the truth values other than 'true' and 'false' if the Liar sentence and the strengthened Liar sentences are not to become paradoxical.

III. NECESSARY CONDITIONS FOR 'THIRD TRUTH VALUE' OR 'TRUTH-VALUE GAPS' THEORIES FOR RESOLVING THE STRENGTHENED LIAR PARADOX

The strengthened Liar sentence 'This sentence is either false or *x*', where '*x*' stands for the 'third truth value' (such as 'paradoxical') or for the words 'occupying a truth-value gap' is paradoxical; for, adopting Mackie's (1973) argument, we find that if one alternative – namely, 'This sentence is false' – is made non-paradoxical by assuming that it is neither true nor false, but is '*x*', then the other alternative, that is, the sentence 'This sentence is *x*', becomes paradoxical. If this alternative, that is, the sentence 'This sentence is *x*', is true, then what it says – namely, 'This sentence is *x*' is *x*, is the case, so 'This sentence is *x*' is '*x*'. On the other hand, if 'This sentence is *x*' is '*x*', then what 'This sentence is *x*' says is the case, so 'This sentence is *x*' is true. So if the strengthened Liar sentence is true, then it is either false or '*x*' (since this is what it says); but if it is false, then this satisfies one of the alternatives, and so makes it true, and if it is '*x*', then this satisfies the other alternative, and equally makes it true.

One way out of this difficulty would be to assume that not merely the sentence 'This sentence is false' but also each of the sentences 'This sentence is *x*' ('This sentence is *x*' is *x*) etc. is '*x*', that is neither true nor false, so that the argument from what any of them says either being or not being the case is blocked. If 'x' stands for 'occupying a truth-value gap', this amounts to saying that the meta-language must itself have the same gap the object language has, and that all these sentences occupy such gaps. As Burge (1979) has said, this would 'manifest a courageous and admirable consistency, but is hardly credible'.

The only other way out is to assume that the sentence 'This sentence is false' is 'x', but the sentence 'This sentence is x' is not 'x', but 'y', where 'y' is another 'third truth value' or a 'different type of truth-value gap' distinguishable from 'x'; that is, there are many different 'third truth values' or 'types of truth-value gaps' x_1, x_2, x_3, etc. and they are so related that if 'This sentence is false' is x_m, then the sentence, 'This sentence is x_m' is never a true sentence, but is 'x_n' where 'x_n' is different from 'x_m', and if for any sentence p, the sentence ('p is x_p' is x_p) is true, then 'p is x_q' is true, where x_q is necessarily different from x_p.

IV. 'TRUE', 'FALSE', AND 'MOST PROBABLY TRUE', ETC.

There are many sentences which can only have some one of the two truth values 'true' and 'false'. Generally, a sentence *describing a physical situation* can only be either true or false; if we do not have sufficient knowledge to ascribe to it its precise truth value, we can at any rate ascribe to it the truth value 'either true or false'.

But not all sentences have some one of the two truth values 'true' and 'false'. The most famous example of a sentence which does not seem to have either of these two truth values is the sentence 'There will be a sea battle tomorrow'. Some philosophers feel that whether it has one of the two truth values 'true' and 'false' depends on whether the future is pre-determined. Many philosophers from the time of Aristotle up until today have felt that this sentence has some third truth value other than 'true' and 'false'; others feel that it has *no* truth value, that is, it occupies a 'truth-value gap'.

Whatever be the correct position regarding the sentence 'There will be a sea battle tomorrow' having a third truth value, or *no* truth value, there is not much doubt that it can be described as being 'most probably true', or 'very probably false', depending on the actual situation today, that is, depending on things like the declaration of war between the countries concerned, the distance between the two navies, the orders issued by their naval headquarters and so on. Furthermore, sentences like

'There will be a sea battle tomorrow' is most probably true are judged to be either true or false on the basis of the actual situation today, and the war offices of the countries concerned are often anxiously considering the truth values of such sentences; on a proper ascription of the truth value 'true' or 'false' to such sentences much war strategy often depends.

Even when a sentence is about a past event, so that we are certain that it has some one of the two truth values 'true' and 'false', our knowledge is often insufficient to ascribe to it any of the two precise truth values. Let p be a sentence which we know is 'either true or false'. Then the sentence 'p is true' can often be described as 'most probably true' etc.; and the resulting meta-sentence ('p is true' is most probably true) is then either true or false; in fact, ascribing the correct truth value from among 'true' and 'false' to such sentences is often of great importance in historical research, or in criminal cases in law courts.

V. SENTENCES WITH TRUTH VALUES OTHER THAN 'TRUE', 'FALSE', AND 'EITHER TRUE OR FALSE'

According to Aristotle, a sentence like 'There will be a sea battle tomorrow' cannot be ascribed either of the truth values 'true' or 'false' (and so cannot be ascribed the truth value 'either true or false' also) because 'it is evident that when in future events there is a real alternative, and a potentiality in contrary directions, the corresponding affirmation and denial have the same character', that is, have a potentiality both of being true and of being false, but are not actually either. Aristotle considered that although neither of its parts is either true or false, the entire disjunction 'Either there will be a sea battle tomorrow or there will not' is definitely true (obviously because the disjunction exhausts all possibilities). As for sentences about past events, the position is different; 'Once it *is*, that which is is-necessarily, and once it is-not, that which is-not necessarily is-not' (*De Interpretatione* Ch.9, quoted in Prior, 1962, p. 241). On this view an assertion p about a past event can have only some one of the two truth values 'true' and 'false', and if there is not enough evidence to enable us to decide which one, we must ascribe to it the truth value 'either true or false'. But when the evidence is not sufficient to enable us to ascribe to p any of the two truth values 'true' and 'false', it may still be sufficient to make the sentence 'p is true' eligible for the truth value 'most probably true'. The position of the sentence 'p is true' is, in this situation, similar to that of the sentence 'There will be a sea battle tomorrow'; the sentence 'p is true' has, then, the potentiality of both being eligible for (with additional evidence) the truth value 'true', and the truth value 'false', but (with the available evidence) is actually eligible for neither of these two truth values. In such situations '*p* is true' can be ascribed only some truth value other than 'true', 'false' and 'either true or false' (that is, it can be ascribed only some truth value like 'most probably true').

Suppose in our presence a coin has been tossed and has fallen flat on the table, but from the distance at which we are standing from the table, we cannot clearly see which side of the coin is facing upwards. In this situation, if somebody says

(A) The result of the toss is heads,

then, clearly (A) is a sentence which is either true or false; in the absence of enough evidence to know which of the two precise truth values 'true' and 'false' is ascribable to it, we can still say that (A) is 'either true or false'. Evidence like 'The coin which was tossed has a strong bias for heads' cannot change this position. We can say

> (B) On evidence that the coin, which has a strong bias for heads has fallen flat on the table, the sentence (A) is 'either true or false'

is a true sentence, and rightly ascribe to (A) the truth value 'either true or false'. If we move closer to the table and have a proper look at the face of the coin, we could ascribe to (A) one of the two precise truth values 'true' and 'false'. With any less conclusive evidence the truth value ascribable to (A) would be the less precise truth value 'either true or false'. Thus whatever be the evidence available, sentence (A) can have only one of the truth values 'true', 'false' and 'either true or false'.

But consider the sentence:

(C) 'The sentence (A) is true'.

Truth values other than 'true', 'false' and 'either true or false' are ascribable to sentence (C) when evidence specified in (B) is available. With evidence specified in (B) we can properly ascribe to sentence (C) the truth value 'most probably true'. If, instead, the evidence available were that the coin which was tossed has a strong bias for tails, the proper truth value ascribable to (C) would have been 'most probably false'. Thus (C) is a sentence which says something about a past event (by asserting a particular truth value for sentence (A)) which can, with appropriate evidence, have truth values 'true' and 'false' (when evidence is adequate to give such truth values to the sentence (A)) or truth values other than 'true' and 'false' (when the evidence available can give to sentence (A) only the truth value 'either true or false') like 'most probably true' etc.

In the above account we consider the truth value of a sentence asserting something about a past event, such as sentence (A), to be 'true', or 'false', or 'either true or false' whatever be the evidence available. On this view if somebody says

> The evidence that the coin which has been tossed and has fallen flat on the table has a very strong bias for heads makes the sentence 'The result of the toss is heads' most probably true,

that would only be an inadequately precise way of saying that the evidence makes (C) most probably true.

Some philosophers consider the meaning of (A) and (C) to be identical and on that ground ascribe to both these sentences the same truth value 'most probably true' on evidence cited in (B). In fact, some arguments leading to very controversial results in probability theory are based on such truth-value ascriptions (Bhave 1989). To show that such ascriptions of truth values involve inconsistency, we must first consider the truth values ascribable to compound sentences 'p and q' (in the sense of p as well as q, and written 'p, q') and 'p or q' on the basis of the truth values of p and of q. The chance of picking up, from a well-shuffled pack of cards, a black card is greater than the chance of picking up a black card which is also a spade; such considerations lead to the following generally accepted rules. If p has the truth value 'true' and q has the truth value 'x-probably-true' (where x is some degree of probability), then 'p, q' is x-probably-true and 'p or q' is true. If p is false, then 'p, q' is false, and 'p or q' has the truth value of q. If p is x_1-probably-true and q is x_2-probably-true, then 'p, q' is z-probably-true, where z is farther away from true than x_1 and than x_2; and 'p or q' is z_1-probably-true, where z_1 is nearer true than the lesser of x_1 and x_2, unless 'p, q' is false, when z_1 is at least as near true as the higher among x_1 and x_2. The acceptance of these intuitively obvious rules is enough to show that sentences like (A) and (C) cannot always have or be ascribed the same truth values.

Suppose we have a heap of coins of two types, one for which the sentence A_{x_1} ('The bias of the coin for heads is x_1') is true, the second for which A_{x_2} ('The bias of the coin for heads is x_2') is true. The proportion of coins of the two types in the heap is z and $(1-z)$. From this heap of coins we select one by a method which ensures to each coin in the heap of coins the same chance of being selected. Let us try to calculate the truth value ascribable to the sentence E ('The selected coin when tossed will fall heads'). E can turn out to be a true prediction either because 'A_{x_1},E' turns out to be true, or 'A_{x_2},E' turns out to be true. So the truth value ascribable to E is the truth value ascribable to the sentence

$$(A_{x_1},E) \text{ or } (A_{x_2},E)$$

given that if A_{x_1} is true, the truth value of (A_{x_1},E) is x_1-probably-true; if A_{x_2} is true, the truth value of (A_{x_2},E) is x_2-probably-true; and that because of the way the coin was selected, the sentence C_{x_1} ('A_{x_1} is true') is z-probably-true and C_{x_2} ('A_{x_2} is true') is $(1-z)$-probably-true.

Since whatever be the method of selection, only one of the sentences A_{x_1} and A_{x_2} can be true, and the other must be false, the truth value ascribable to each is 'either true or false' with the stipulation that if A_{x_1} is true, then A_{x_2} is false, and if A_{x_1} is false, then A_{x_2} is true. With such ascription of truth values the sentence '(A_{x_1},E) or (A_{x_2},E)' has the truth value either 'x_1-probably-true' or 'x_2-probably-true' in agreement with our intuition that one of these two truth values must hold for E in this situation, since the coin selected has some one of the biases x_1 and x_2.

However, if we adopt for A_{x_1} and A_{x_2} the truth values of C_{x_1} and C_{x_2} respectively, we get for '(A_{x_1},E) or (A_{x_2},E)' a truth value depending on x_1, x_2 and z and $(1-z)$ which need not even agree either with 'x_1-probably-true' or with 'x_2-probably-true' and which, moreover, can change with change of z, that is, with the constitution of the heap from which the coin was selected. It is as if the coin, on selection, changes its bias for heads, depending on the constitution of the heap from which it was selected. Clearly, adopting the same truth value for A_{x_1} and C_{x_1} leads to inconsistency.

As another instance, suppose samples of size N are prepared from a parent population of individuals of two types. If an equal number of samples of each type (corresponding to the number r of individuals of the first type in the sample) are prepared, and from them one sample is selected by a method which ensures to each sample the same chance of being selected, then of this sample the $(N+1)$ sentences $\bar{r}$ ('The sample selected has r members of the first type') are such that each of them has the truth value 'either true or false' with the stipulation that one of them is true and all the others are false. But the $(N+1)$ sentences '$\bar{r}$ is true' corresponding to $r = 0,1,2,\ldots N$ are such that every one of them has the same truth value x-probably-true, where x is some degree of probability. The sentences $\bar{r}$ and '$\bar{r}$ is true' do not have the same truth value for any value of r from 0 to N.

In the following sections we shall examine the logical principles that connect the truth values of sentences like (C), '(C) is most probably true', ' "(C) is most probably true" is most probably true' etc.

VI. SENTENCES FORMED BY ADDING THE WORDS 'IS TRUE' OR 'IS FALSE' TO SENTENCES WITH TRUTH VALUES OTHER THAN 'TRUE' AND 'FALSE'

When the available evidence is only that the coin which has been tossed and has fallen flat on the table has a strong bias for heads, the sentence (C)

(C) 'The result of the toss is heads' is true

has the truth value 'most probably true'. If in this situation somebody were to say 'Sentence (C) is true', that would only be a false sentence, since the correct truth value of (C) is (with the available evidence) not 'true', but only 'most probably true'. Thus sentence (D)

(D) (C) is true

has the truth value 'false'.

Similarly the sentence formed by adding the words 'is false' to sentence (C), namely, the sentence (E)

(E) (C) is false

has the truth value 'false'.

Clearly, the sentences formed by adding 'is false' to (D) and to (E) are both true sentences. Thus sentences

(F) (C) is true is false
(G) (C) is false is false

are both true sentences. But the sentences formed by replacing 'is true is false' in (F) by 'is false' and by replacing 'is false is false' in (G) by 'is true' are both false.

In the context of the evidence mentioned above, the replacement of 'is true is false' by 'is false', and of 'is false is false' by 'is true' in true sentences does not lead to true sentences. In another context, when the evidence is such that (C) has a truth value 'true' or 'false', such substitutions in true sentences would lead to true sentences.

VII. SENTENCES ENDING IN 'IS MOST PROBABLY TRUE, IS MOST PROBABLY TRUE'

The truth value of a sentence (C) ('The result of the toss is heads' is true) is 'most probably true' when the evidence – namely, the sentence 'The coin has a very strong bias for heads' – is known to be true. We can look upon the sentence (C_1)

(C_1) (C) is most probably true

as the inference drawn from the sentence 'The coin has a very strong bias for heads'. If the sentence 'The coin has a very strong bias for heads' is known to be only 'most probably true' (as when all that is known is that the coin was chosen at random from a heap of coins most of which had a strong bias for heads), then the truth value of the inference (C_1) would itself be only 'most probably true'; and the truth value of the sentence (C) would be something less, something like 'very probably true'. Thus if the sentence (C_2)

(C_2) (C_1) is most probably true

is true, the sentence

(C) is very probably true

is true, and has the truth value 'true'.

Such reasoning is in universal use in practical life and in scientific research. In general if x_1,x_2,x_3 are different degrees or ranges of degrees of probability, we find that if for any sentence (X),

'(X) is x_1-probably-true' is x_2- probably-true

is true, then

(X) is x_3-probably-true

is true, where x_3 is a different degree or range of degrees of probability from x_1. This holds also when x_2 is the same degree of probability as x_1. This rule obviously holds for expressions like 'most probably false', 'very probably false', etc.

In the example discussed above, x_3 was a lesser degree of probability than x_1; but in some situations (as when (C) is replaced by (C′) ('The result of the toss is tails' is true)) x_3 would be greater than x_1. What always holds is that x_3 is different from x_1.

Thus the truth values 'x_1-probably-true' etc. (and 'x_1-probably-false' etc.) satisfy the relations stated at the end of section III, that is, the relations which they must satisfy if the paradox of the Liar sentence and the strengthened Liar sentence is to be resolved on the assumption of there being several 'third truth values', all different from 'true' and 'false'.

VIII. THE LIAR AND THE STRENGTHENED LIAR SENTENCES FROM THE POINT OF VIEW OF MANY VALUED LOGIC

From the point of view of many-valued logic incorporating truth values like 'x-probably-true' etc. in addition to 'true' and 'false', and the logical principles governing inferences involving such truth values, the Liar sentence (S) ('This sentence is false') must have a truth value like 'x-probably-true'. For each of the hypotheses ('(S) is true') and ('(S) is false') leads to contradiction and paradox, showing that both of these are false, leading to the conclusion that (S) is x-probably-true, where x is some degree of probability. As for the strengthened Liar sentence ($\bar{S}$) (($\bar{S}$): 'This sentence is x-probably-true'), the hypothesis that ($\bar{S}$) is true leads to contradiction; for if ($\bar{S}$) is true, what it says is the case, that is, '($\bar{S}$) is x-probably-true' is true, that is, ($\bar{S}$) has a truth value other than 'true'. The hypothesis that ($\bar{S}$) has the truth value 'x-probably-true' also leads to contradiction; for if ($\bar{S}$) has the truth value 'x-probably-true', then what ($\bar{S}$) says is the case, so ($\bar{S}$) must have the truth value 'true'. So ($\bar{S}$) can have either (1) a truth value 'y-probably-true' where y is different from x, or (2) the truth value 'false'. But if ($\bar{S}$) is false, what it says is not the case, that is, '($\bar{S}$) is x-probably-true' is false, so ($\bar{S}$) can either be true (which, as we have already seen, is impossible) or has a truth value like 'y-probably-true', where y is different from x. For similar reasons, the sentence 'This sentence is y- probably-true' must have a truth value 'z-probably-true', where z is different from both x and y. Since this process can be repeated indefinitely, the number of degrees of probability x,y,z . . . . must be infinite. But in natural languages an infinite number of degrees of probability do exist; in fact, in quantum physics the different degrees of probability in use to describe physical phenomena can be put in one-to-one correspondence with the real numbers in the range 0 to 1.

About the degrees of probability x,y,z, . . . etc. we cannot say anything more than that different degrees of probability, that is, the truth values 'x-probably-true', 'y-probably-true', etc. are all different from each other and from 'true' and 'false'. This is analogous to the situation when, for lack of sufficient knowledge, a sentence has to be allotted the truth value 'either true or false'. That the truth values of (S) and ($\bar{S}$) are different is natural, since their meanings are different.

The Liar sentence is comparable to sentences which, because of their form, must have some particular truth value. Thus just as if *p* is any sentence having some one of the two truth values 'true' and 'false', the sentence '*p* and not-*p*' must be false, and the sentence 'either *p* or not-*p*' must be true, similarly the Liar sentence must have, because of its form, a truth value like '*x*-probably-true' or '*x*-probably-false', and the sentence 'This sentence is *x*-probably-true' must have a truth value like '*y*-probably-true'. This is because the Liar sentence

1. refers to no other sentence;
2. says nothing other than asserting its own truth value;
3. asserts the truth value 'false'.

(Unlike the Liar sentence, 'This sentence is true' poses no paradox.)

IX. 'MOST PROBABLY TRUE' ETC. AS TRUTH VALUES

Whether expressions like 'most probably true' etc. can properly be called 'truth values' like 'true' and 'false' is a matter on which philosophers hold different views. Those who have attempted to develop a logic of the probability of propositions deem such expressions to be truth values, and consider 'true' and false' as the limits of 'most probably true' and 'most probably false'. Some feel it is misleading to 'think of probability values as lying on a scale whose extremes stand for truth and falsehood' (Mackie 1973). About Lukasiewicz's three truth values Kneale and Kneale (1984) have stated that 'they do not appear to be truth values, nor yet pure modal values, but rather certainty values'.

In calling expressions like 'most probably true' etc. truth values I have followed the practice of those who have attempted to develop a logic for the probability of propositions. But the argument in this paper is not dependent on any particular view as to whether such expressions can most appropriately be called 'truth values' or 'certainty values' or 'truth-value gaps', or whether 'true' and 'false' can properly be considered identifiable with the limits of 'most probably true' and 'most probably false'. For the purposes of this paper, expressions like 'most probably true' etc. are truth values only in the sense that it is meaningful to describe sentences as 'most probably true' etc., and such descriptions can be 'true' or 'false'; and that logical principles actually in use in practical life and scientific research govern the rules for making inferences and for deriving true sentences from other true sentences, even when the sentences in question contain expressions like 'most probably true' etc. The logical principles on which the argument in this paper is based are the weakest such principles commanding the most widespread acceptance; the stronger (and controversial) principles and axioms used by many philosophers who have developed a logic for the probability of propositions play no part in the argument developed in this paper.

For our present purposes, far more important than the question of whether expressions like 'most probably true' should properly be termed 'truth values' or 'different ways of filling truth-value gaps' is the question of whether there are any other expressions which could more illuminatingly be termed 'third truth values' or 'truth-value gaps' as solutions of the paradox posed by the Liar and the strengthened Liar sentences.

X. CONCLUSION

Several writers (among them Martin 1984) have pointed out that there cannot be a unique solution to the Liar paradox; and Thomson (1962) has remarked that the importance of the paradoxes lies 'rather in the conclusions they give rise to'. It would seem that the conclusion that the Liar and the strengthened Liar sentences lead to is that any theory which aims at explicating the univocality of 'true', and at accounting for, rather than merely obstructing, paradoxical reasoning (Burge 1979), and at a proper analysis of our intuitions about 'true' (Kripke 1975), must grapple with expressions like 'most probably true' etc. in vogue in natural languages, and the logical principles governing their use in describing sentences and making inferences.

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